FORM PTO-1390 (Modified) (REV 11-2000) U.S. DEPARTMENT OF COMMERCE PATENT AND TRADEMARK OFFICE	ATTORNEY'S DOCKET NUMBER
TRANSMITTAL LETTER TO THE UNITED STATES	A-9185
DESIGNATED/ELECTED OFFICE (DO/EO/US) CONCERNING A FILING UNDER 35 U.S.C. 371	U.S. APPLICATION NO. (IF KNOWN, SEE 37 CFR **09/889670**

INTERNATIONAL APPLICATION NO. **PCT/IB00/00212**	INTERNATIONAL FILING DATE **19 January 2000**	PRIORITY DATE CLAIMED **19 January 1999**

TITLE OF INVENTION

MULTIEXPONENTIAL SIGNAL PROCESSING METHOD AND APPARATUS USING LINEAR RESOLUTION

APPLICANT(S) FOR DO/EO/US

COVER, Keith

Applicant herewith submits to the United States Designated/Elected Office (DO/EO/US) the following items and other information:

1. ☒ This is a **FIRST** submission of items concerning a filing under 35 U.S.C. 371.

2. ☐ This is a **SECOND** or **SUBSEQUENT** submission of items concerning a filing under 35 U.S.C. 371.

3. ☐ This is an express request to begin national examination procedures (35 U.S.C. 371(f)). The submission must include items (5), (6), (9) and (24) indicated below.

4. ☒ The US has been elected by the expiration of 19 months from the priority date (Article 31).

5. ☒ A copy of the International Application as filed (35 U.S.C. 371 (c) (2))

    a. ☒ is attached hereto (required only if not communicated by the International Bureau).

    b. ☒ has been communicated by the International Bureau.

    c. ☐ is not required, as the application was filed in the United States Receiving Office (RO/US).

6. ☐ An English language translation of the International Application as filed (35 U.S.C. 371(c)(2)).

    a. ☐ is attached hereto.

    b. ☐ has been previously submitted under 35 U.S.C. 154(d)(4).

7. ☒ Amendments to the claims of the International Application under PCT Article 19 (35 U.S.C. 371 (c)(3))

    a. ☐ are attached hereto (required only if not communicated by the International Bureau).

    b. ☐ have been communicated by the International Bureau.

    c. ☐ have not been made; however, the time limit for making such amendments has NOT expired.

    d. ☒ have not been made and will not be made.

8. ☐ An English language translation of the amendments to the claims under PCT Article 19 (35 U.S.C. 371(c)(3)).

9. ☒ An oath or declaration of the inventor(s) (35 U.S.C. 371 (c)(4)).

10. ☐ An English language translation of the annexes of the International Preliminary Examination Report under PCT Article 36 (35 U.S.C. 371 (c)(5)).

11. ☒ A copy of the International Preliminary Examination Report (PCT/IPEA/409).

12. ☒ A copy of the International Search Report (PCT/ISA/210).

Items 13 to 20 below concern document(s) or information included:

13. ☐ An Information Disclosure Statement under 37 CFR 1.97 and 1.98.

14. ☐ An assignment document for recording. A separate cover sheet in compliance with 37 CFR 3.28 and 3.31 is included.

15. ☐ A **FIRST** preliminary amendment.

16. ☐ A **SECOND** or **SUBSEQUENT** preliminary amendment.

17. ☐ A substitute specification.

18. ☐ A change of power of attorney and/or address letter.

19. ☐ A computer-readable form of the sequence listing in accordance with PCT Rule 13ter.2 and 35 U.S.C. 1.821 - 1.825.

20. ☒ A second copy of the published international application under 35 U.S.C. 154(d)(4).

21. ☐ A second copy of the English language translation of the international application under 35 U.S.C. 154(d)(4).

22. ☐ Certificate of Mailing by Express Mail

23. ☒ Other items or information:

    Copy of Form PCT/IB/308
    10 sheets of formal drawings

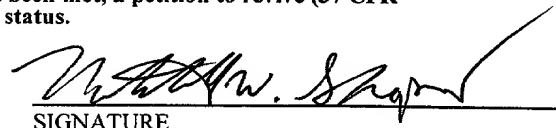| U.S. APPLICATION NO. (IF KNOWN, SEE 37 CFR) **09/889670**	INTERNATIONAL APPLICATION NO. PCT/IB00/00212	ATTORNEY'S DOCKET NUMBER A-9185

24. The following fees are submitted:.	CALCULATIONS PTO USE ONLY

BASIC NATIONAL FEE (37 CFR 1.492 (a) (1) - (5)) :

☐ Neither international preliminary examination fee (37 CFR 1.482) nor international search fee (37 CFR 1.445(a)(2)) paid to USPTO and International Search Report not prepared by the EPO or JPO **$1000.00**

☒ International preliminary examination fee (37 CFR 1.482) not paid to USPTO but International Search Report prepared by the EPO or JPO **$860.00**

☐ International preliminary examination fee (37 CFR 1.482) not paid to USPTO but international search fee (37 CFR 1.445(a)(2)) paid to USPTO **$710.00**

☐ International preliminary examination fee (37 CFR 1.482) paid to USPTO but all claims did not satisfy provisions of PCT Article 33(1)-(4) **$690.00**

☐ International preliminary examination fee (37 CFR 1.482) paid to USPTO and all claims satisfied provisions of PCT Article 33(1)-(4) **$100.00**

ENTER APPROPRIATE BASIC FEE AMOUNT =	**$860.00**

Surcharge of **$130.00** for furnishing the oath or declaration later than ☐ 20 ☐ 30 months from the earliest claimed priority date (37 CFR 1.492 (e)). — **$0.00**

CLAIMS	NUMBER FILED	NUMBER EXTRA	RATE		
Total claims	21 - 20 =	1	x $18.00	$18.00	
Independent claims	4 - 3 =	1	x $80.00	$80.00	
Multiple Dependent Claims (check if applicable).			☐	$0.00	

TOTAL OF ABOVE CALCULATIONS =	$958.00	

☐ Applicant claims small entity status. (See 37 CFR 1.27). The fees indicated above are reduced by 1/2. — **$0.00**

SUBTOTAL =	$958.00	

Processing fee of **$130.00** for furnishing the English translation later than ☐ 20 ☐ 30 months from the earliest claimed priority date (37 CFR 1.492 (f)). + **$0.00**

TOTAL NATIONAL FEE =	$958.00	

Fee for recording the enclosed assignment (37 CFR 1.21(h)). The assignment must be accompanied by an appropriate cover sheet (37 CFR 3.28, 3.31) (check if applicable). ☐ — **$0.00**

TOTAL FEES ENCLOSED =	$958.00	
Amount to be: refunded	$	
charged	$	

a. ☒ A check in the amount of ____ **$958.00** ____ to cover the above fees is enclosed.

b. ☐ Please charge my Deposit Account No. _____ in the amount of _____ to cover the above fees. A duplicate copy of this sheet is enclosed.

c. ☒ The Commissioner is hereby authorized to charge any additional fees which may be required, or credit any overpayment to Deposit Account No. ____ **50-1165** ____ A duplicate copy of this sheet is enclosed.

d. ☐ Fees are to be charged to a credit card. **WARNING: Information on this form may become public. Credit card information should not be included on this form.** Provide credit card information and authorization on PTO-2038.

NOTE: Where an appropriate time limit under 37 CFR 1.494 or 1.495 has not been met, a petition to revive (37 CFR 1.137(a) or (b)) must be filed and granted to restore the application to pending status.

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31,568
REGISTRATION NUMBER

July 19, 2001
DATE

F-9185-PCT

Title: Multiexponential Signal Processing
Method and Apparatus Using Linear Resolution

Inventor: Keith S. Cover

SPECIFICATION

CROSS-REFERENCE TO RELATED APPLICATION

This application claims the benefit of Provisional
Application No. 60/116,362 filed January 19, 1999,
incorporated herein by reference.

5   BACKGROUND OF THE INVENTION

This invention is concerned with signal processing of
transients, and more particularly, with multiexponential
signal processing.

In the recent REVIEW article entitled "Exponential

10   analysis in physical phenomena", Review of Scientific
Instruments, Vol. 70, No. 2, Feb. 1999, pages 1233-1257
(incorporated herein by reference), authors Andrei A.
Istratov and Oleg F. Vyvenko consider various aspects of
multiexponential analysis. It is apparent from the text of

15   the REVIEW article, and from the over 300 literature
citations, that exponential analysis in physical phenomena
is a subject of considerable interest in many scientific and
technological disciplines, including, inter alia, solid

state physics, medicine, biology and biophysics, geophysics, optics, engineering, chemistry and electro-chemistry.

As pointed out in the REVIEW article, many physical phenomena are described by first-order differential

5 equations whose solution is an exponential decay. The amplitude and time constant of the exponential decay carry information about the nature of the phenomenon being studied. As commonly happens, a number of exponential processes take place simultaneously, and equipment employed

10 in analyzing such exponential processes (multiexponential decays) yields a signal which is a sum of a plurality of exponential components.

Obtaining useful information from multiexponential decays is not a simple task. Many methods of

15 multiexponential analysis and many algorithms for this purpose have been proposed, but all of them have limitations.

U.S. Patent No. 5,517,115 issued May 14, 1996 to Manfred G. Prammer (incorporated herein by reference)

20 proposes a method and apparatus for efficient processing of nuclear magnetic resonance (NMR) echo trains in well logging. A priori information about the nature of the expected signals is used in an attempt to obtain an approximation of a model using a set of pre-selected basis

25 functions. A singular value decomposition (SVD) is applied to a matrix incorporating information about the basis functions, and is stored off-line in a memory. During

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actual measurement, the apparatus estimates a parameter
related to the signal-to-noise ratio (SNR) of the received
NMR echo trains and uses it to determine a signal
approximation model in conjunction with the SVD of the basis
function matrix. This approximation is used to determine,
in real time, attributes of the earth formation being
investigated.

Techniques such as those described in the Prammer
patent rely heavily upon a priori information about the
nature of the expected signals. In attempting to obtain a
reliable estimate of an unknown model, i.e., parameters of a
subject being investigated, such techniques rely on one or
more numerical algorithms for multiexponential analysis
which use so-called "fitting routines" in an attempt to fit
data to the model.

Underlying the present invention is the discovery that
substantially improved results can be attained without using
fitting routines, and, instead, by emphasizing better linear
resolution. As pointed in the REVIEW article, fitting
routines work well only if the hypothesis of the number of
multiexponential components is correct and an initial
approximization is close to the true solution. The present
invention, which does not use fitting routines, avoids the
problems that are inherent in the use of such routines.

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BRIEF DESCRIPTION OF THE INVENTION

An object of the present invention is to provide improved methods and apparatus for the analysis of transients and for obtaining useful information therefrom.

More particularly, an object of the invention is to provide improved multiexponential signal processing.

One aspect of the invention involves a computer-readable medium containing a set of coefficients that define a transform operator such as a matrix.

Another aspect of the invention involves a method of calculating a transform operator utilizing a plurality of resolution functions.

Another aspect of the invention involves a method of multiexponential signal processing in which multiexponential signals are sampled, and the above-mentioned transform operator is applied to the sampled signals.

Another aspect of the invention involves an apparatus for multiexponential signal processing that comprises a signal processor including the above-mentioned transform operator.

The present invention starts with the construction of appropriate transform operators. Once appropriate transform operators have been constructed, they are incorporated in signal processors of analytical instrumentation for processing data. Generally, such instrumentation includes a computer, as is well known in multiexponential analysis. Multiexponential data signals are sensed or detected by

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conventional equipment and are input to the transform

operator of the computer for signal processing. The signals

may be applied in real time or they may be read out from a

suitable storage medium.

5   Typically, the signals are applied to the transform

operator in digital form after conventional sampling and

analog-to-digital conversion. For example, digital samples

of multiexponential decays may be obtained at equally-spaced

instants in time, beginning at or just after the start of a

10   multiexponential signal.

The invention makes use of linear resolution to obtain

a better estimate of an unknown model. The present invention

provides the very desirable property of optimal linear

resolution of the unknown model when plotted against the log

15   of the time constant of the decay curves.

BRIEF DESCRIPTION OF THE DRAWINGS

The invention will be further described in conjunction with the accompanying drawings, which illustrate preferred (best mode) embodiments, and wherein:

Fig. 1 is a diagram showing a matrix with coefficients a_{11} . . . a_{NK} for transforming data d_1 . . . d_N to produce parameters m_1 . . . m_K of an estimate of an unknown model;

Fig. 2 is a flow chart showing a method for calculating the coefficients of a row of a transform matrix in accordance with the invention;

Fig. 3 is a table showing, <u>inter alia</u>, coefficients for three rows of a matrix, for τ's of interest, namely 1, 10, 100;

Fig. 4 is a diagram showing data functions, resolution functions, noise response, and point spread functions for a matrix constructed in accordance with the invention where the noise gain (NG) is 1.0000;

Fig. 5 is a similar diagram for another matrix constructed in accordance with the invention for a noise gain of 3.1623;

Fig. 6 is a diagram showing resolution functions for four matrices constructed in accordance with the invention with different noise gains;

Fig. 7 is a diagram showing point spread functions associated with four matrices constructed in accordance with the invention for different noise gains;

Fig. 8 is a block diagram showing apparatus for processing data in accordance with the invention;

Fig. 9 is a diagram showing decay curves of an MRI; and

Fig. 10 is a diagram showing MRI relaxation

5   distributions for four matrices constructed in accordance with the invention with different noise gains.

DETAILED DESCRIPTION OF THE INVENTION

One of the principal objectives of the present invention is to provide signal processing of transients,

10   such as multiexponential decays, producing outputs that are better estimates of an unknown model to be investigated. Such estimates permit better interpretation of data, so that a user (researcher, physician, scientist, or engineer, for example) can obtain more accurate information as to the

15   nature of the unknown model. Considered from one point of view, the invention may be looked upon as a better digital lens that provides improved resolution, just as a better optical lens provides improved resolution.

A first step in achieving the objectives of the

20   invention is to construct an improved transform operator, conveniently in the form of a matrix. The manner in which a transform matrix may be constructed, pursuant to the invention, will be described in detail later. The actual transform operator will depend upon its intended

25   application, for example, medical imaging or well logging. For any application, several different transform operators

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may be constructed, to provide a user with greater flexibility.

The present invention requires an understanding of what is referred to in the art as estimating a solution of a linear inverse problem. The linear inverse problem is one of communicating to an interpreter what is known and what is not known about an unknown model. The almost universal practice in the prior art for estimating the solution of a linear inverse problem is to calculate one or more of an infinite number of estimates of the model which fit the data, i.e., which reproduce the data to within the noise that is present. Whenever possible, a priori information is used to choose which of the estimates to calculate. However, a priori information may not be sufficiently available or may be suspect.

In applying the present invention to multiexponential decays, an estimate of an unknown relaxation distribution (model) is obtained by linearly resolving each point of the unknown relaxation distribution as precisely as possible within the limits of the noise. In general, such estimates do not reproduce the data. Rather, they are obtained by optimizing the linear resolution in a manner that will be described later.

The term "linear" used herein in connection with "resolution" has the following meaning:

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Consider a transform operator A which transforms functions (or vectors) x1 and x2 to y1 and y2, respectively. As equations, this is stated:

A x1 = y1 and A x2 = y2.

The transform operator A is defined as "linear" if for any two real numbers a and b

A(a x1 + b x2) = a y1 + b y2.

A transform operator which is a matrix will always have linear resolution. It is possible to construct non-matrix transform operators which behave similarly to matrices for only a restricted set of functions (or vectors). Moreover, a matrix can be derived from a non-matrix transform operator that expresses this behavior.

The present invention involves the following relationship between a true model $m^T(y)$ and data d_k (e.g., multiexponential decays):

$$d_k = \int_I m^T(y)\, g_k(y)\, dy \qquad (1)$$

where I is the appropriate domain of definition and $g_k(y)$ represents a data function, more particularly, one of the N data functions which compose the data kernel of a linear transform. A data function is a mathematical representation of how a model is mapped to a particular data point. Equation (1) is referred to as "the forward problem."

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Decaying systems generally have a point, usually defined as t=0 (where t is time), at which a system begins decaying. The system decays to a constant value, often zero, as t approaches infinity. In nuclear magnetic resonance (NMR), for example, the t=0 point is when an excitation pulse energizes a sample.

The multiexponential forward problem commonly used in data analysis and inverse theory has the form

$$d_k = \sum_i m_i e^{-t_k/\tau_i} \qquad (2)$$

In this equation d_k represents the data; m_i represents the unknown model, and e^{-t_k/τ_i} represents the data function, where τ designates the time constant of the exponential decay.

Equation (2) can be expressed in the continuous form

$$d_k = \int_0^\infty \sum_i m_i \delta(\tau - \tau_i) e^{-t_k/\tau} \qquad (3)$$

where $\delta()$ is the Dirac delta function.

Incidentally, as used herein, the range of indices such as i, j and k are determined by the equation in which they are used. For example in the equation

$$\bar{R}(y) = \sum_i b_i g_i(y) \qquad (4)$$

10

the index i would be assumed to range over all the data

functions $g_i(y)$ · which are normally numbered with positive

integers starting at 1. But in the equation

$$NG_i = \sqrt{\sum_j a_{ij}}$$ (5)

the index i ranges over the rows of a transform matrix and

the index j ranges over the columns of the transform matrix,

which is the same range as the data functions in the

previous equation.

It is desirable to plot the relaxation distribution

(unknown model) versus the logarithm of τ, and to determine

the resolution of a relaxation distribution on a log scale.

Applying a change of variables $y=\ln(\tau)$ to equation (3),

without simplification, yields

$$d_k = \int_{-\infty}^{+\infty} \sum_i m_i \delta(e^y - e^{y_i}) e^{-t_k e^{-y}} (e^y dy)$$ (6)

Applying the Dirac delta function identity

$$\delta(f(x)) = \sum_i \frac{1}{f'(x_i)} \delta(x - x_i) \text{ for each } f(x_i) = 0$$ (7)

to equation (6) along with standard simplification yields

$$d_k = \int_{-\infty}^{+\infty} \sum_i m_i \delta(y - y_i) e^{-t_k e^{-y}} dy$$ (8)

Substituting

$$m^T(y) = \sum_i m_i \delta(y - y_i)$$ (9)

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into equation (8) gives

$$d_k = \int_{-\infty}^{+\infty} m^T(y) \, e^{-t_k e^{-v}} \, dy.\tag{10}$$

where $m^T(y)$ represents the unknown model, and $e^{-t_k e^{-v}}$ represents the data function, which may be expressed

5 as:

$$g_k(y) = e^{-t_k e^{-v}}.\tag{11}$$

Hereinafter, the form of the multiexponential transform in equation (10) will be termed the forward problem.

It should be noted that the data function approaches 1
.0 as τ approaches infinity for all values of t_k. Pursuant to the invention, the data function of equation (11) has been found to be very useful in multiexponential analysis, but other data functions may be appropriate for other
15 applications of the invention.

As stated earlier, a first step in achieving the objectives of the invention is to construct a transform operator, such as a transform matrix, which maps data (e.g. decay signals) to unknown model parameters. Fig. 1 is a
20 diagram showing a matrix with coefficients $a_{11} \ . \ . \ . \ a_{iK}$ for transforming data $d_1 \ . \ . \ . \ d_N$ to produce parameters $m_1 \ . \ . \ . \ m_k$ of an unknown model. The transform matrix is typically a matrix in which each row corresponds to a τ of interest. Selection of τ's of interest and data points for initial
25 coefficients will be guided by available information in the particular field in which the matrix is to be used.

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In accordance with the invention, it is desired to obtain an estimate of an unknown model with linear resolution, and since the data functions used are linear functions of the model, the estimate of the unknown model

5   can be calculated by multiplying the data by a matrix. The matrix is chosen so that each point of the estimate of the unknown model linearly resolves the corresponding point of the model as well as possible with an acceptable noise gain i.e., with optimal linear resolution. Since matrix

10   multiplication is a linear operation, it yields an estimate which does not necessarily reproduce the data, but which does have linear resolution. Linear resolution is a desirable property of an estimate, because each point of the estimate resolves the corresponding point of the model in

15   the same way, independent of any particular model.

A goal in constructing a transform matrix in accordance with the invention is to calculate linear combinations of the data functions which yield a resolution function that resolves as small a region of the unknown model as possible.

20   Accordingly, it is preferred that each resolution function corresponding to a row of the matrix be characterized by optimal linear resolution. Preferred criteria for selecting coefficients of a transform matrix which yield optimal linear resolution are described later. Together, the

25   resolution function and its noise gain give a concise formulation of the ambiguity of a point in an estimate of an

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unknown model from information given by data and
corresponding data functions.

Calculation of a transform matrix proceeds row by row.
It is convenient to use the same noise gain for all rows of
a transform matrix, but this is not required. As noted
above, each row normally corresponds to a τ of interest.
The rows can be ordered by increasing τ from top to bottom
of the matrix. A spacing of 16 τ values per decade has been
found to work well. As an example, τ values may be
calculated between $0.1t_{min}$ and $10t_{max}$, where t_{min} is the
smallest time at which a decay signal is to be sampled and
t_{max} is the largest time. Each row of the matrix corresponds
to a resolution function that is centered on a τ of
interest.

It is presently believed that the best way to calculate
the coefficients of a particular transform matrix (which,
incidentally, may have only one row) is to solve a
constrained minimization problem.

The information needed before the constrained
minimization can be performed are (1) the data function,
$g_i(y)$, for each data point, d_i, (2) the standard deviation of
the expected noise, σ_i, of each data point, (3) the desired
noise gain, NG, and (4) the τ of interest, τ_k, on which the
resolution function is to be centered.

The constrained minimization problem to be solved is to
minimize I in the equation:

$$I = \int_{-\infty}^{+\infty} \bar{R}(y)dy \qquad (12)$$

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by varying b_i, where

$$\bar{R}(y) = \sum_i b_i g_i(y) \qquad (4)$$

5   and where b_i are trial coefficients of a transform matrix

row and $\bar{R}(y)$ is a trial resolution function. More

information on resolution functions can be found in Parker,

Robert L. 1994; <u>Geophysical Inverse Theory</u>; Princeton, New

Jersey; Princeton University Press, incorporated herein by

10   reference.

The trial resolution function preferably complies

substantially with the following constraints:

(i) $\bar{R}(y) \geq 0$ [Nonnegative constraint]

(ii) $\bar{R}(y_k) \geq 1$ [Peak constraint]

15   (iii) $\bar{R}(y)$ monotonically decreases from y_k [Monotonicity

constraint]

(iv) $\sqrt{\sum_{i=1}^{N} b_i^2 \frac{\sigma_i^2}{\sigma_1^2}} \leq NG \int_{-\infty}^{+\infty} \sum_{i=1}^{N} b_i g_i(y) dy$ [Noise gain constraint]

Constraint (iii) may be satisfied automatically for the

data function used for the multiexponential problem. Thus,

20   while it is stated as a separate constraint, it is to be

understood that it may, in fact, be satisfied inherently.

To arrive at the final coefficients, a_{ij}, for row i of

the transform matrix, once the trial coefficients b_i which

minimize I have been found, it is highly desirable that the

25   trial coefficients be scaled by a constant so that the area

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of their associated resolution function is unity on a log scale. This is accomplished by

$$a_{ij} = b_j / \int_{-\infty}^{+\infty} \sum_{k=1}^{N} b_k g_k(y) dy \qquad (13)$$

Making the area of the resolution function unity insures that each point in the estimate of the unknown model is a local average.

Incidentally, for a data point d_j, sampled on a decay curve at time t_j, the data function previously described can be modified by a balancing function $B(y)$, so that the data function becomes

$$g_k(y) = \frac{e^{-t_k e^{-y}}}{B(y)} \qquad (14)$$

This modification of the data function has the effect of making the resulting transform matrix generate an estimate of the unknown model multiplied by $B(y)$ but with the required noise gain. While many balancing functions are possible, a useful balancing function is $B(y) = e^{+wy}$. The choice w = 1 may be useful since it increases the amplitudes of the relaxation components at late time constants by a factor of τ.

Constructing a transform matrix in which each resolution function corresponding to a row of the matrix is characterized by optimal linear resolution preferably involves a process termed constrained optimization. In the preferred embodiment of the invention, this process includes an outer loop, a middle loop and an inner loop. The outer

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loop converts the constrained optimization problem to a series of unconstrained multidimensional minimization problems using the simple penalty method described by Fletcher (Fletcher, R. 1987 _Practical Methods of Optimization_; Toronto; John Wiley & Sons) incorporated herein by reference. The middle loop converts the unconstrained multidimensional minimization problems into a series of one dimensional (1D) minimization problems via the conjugate gradient method described by Press et al. (Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P. 1992; _Numerical Recipes in C: The art of scientific computing_; 2nd Ed.; Cambridge UK; Cambridge University Press) incorporated herein by reference. The 1 dimensional minimization problems are solved using the golden section search or parabolic interpolation described by Press et al.

A desirable condition for each resolution function is that its area should be substantially minimized. Calculating the area of the trial resolution function I to be minimized, given the trial coefficients, b_i, can be accomplished using the equation:

$$I = \sum_i b_i A_i \qquad (15)$$

where A_i is the area of the data function, $g_i(y)$, given by the equation

$$A_i = \int_{-\infty}^{Y1} g_i(y) dy \qquad (16)$$

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The integral's upper limit of infinity has been replaced by the value Y1, which is a large finite number. Experience has shown that Y1 = ln(1000*t_{max}), is a good choice.

The simple penalty method converts the constrained optimization problem to an unconstrained one via residues, r_m. A residue is a measure of how much a particular constraint is violated. If a particular residue is greater than zero, then the corresponding constraint is considered satisfied. The more negative the residue, the more the constraint is violated. The constrained optimization problem then becomes the unconstrained one of minimizing I^P in the equation:

$$I^P = \sum_i b_i A_i + P \sum_m min(r_m, 0)^2 \qquad (17)$$

where min(x,y) returns the minimum and x and y. The value of P starts small, and is then increased by a small factor, for example, 0.1, after each unconstrained minimization. The solution to one constrained minimization is used as a starting point for the next constrained minimization. The value of P is increased until I^P has stabilized. The test for stability is whether the value of I^P changes by an accumulative factor of 10^{-6} over four consecutive increases in P.

Each of the non-negative, peak, monotonicity and noise gain constraints will usually have associated residues. Before the residues can be calculated, the data functions and the resolution functions are discretized.

The data functions and the resolution functions are discretized by sampling them at e.g., 16 points per decade of τ starting at $\tau = 0.01$ time units and continuing to $\tau = 1000*t_{max}$ time units. The discretized data functions, $g_i(y)$, are represented as g_{il}. The total number of points at which the functions are sampled is denoted L. The discretized trial resolution function is represented by $\bar{R}_l$.

Thus

$$\bar{R}_l = \sum_i b_i g_{il} \qquad (18)$$

The residues of the nonnegative constraint, r_l^{NN}, are defined to be

$$r_l^{NN} = \bar{R}_l \qquad (19)$$

giving L residues.

The residues of the peak constraint, r^P, are defined to be

$$r^P = \bar{R}_{l_P} - 1 \qquad (20)$$

where l_P is the index of the discretized trial resolution function at the τ of interest. This yields one residue.

The residues of the monotonicity constraint are defined to be

$$r_l^{ML} = \bar{R}_l - \bar{R}_{l-1} \quad 2 \geq l \geq l_P \qquad (21)$$

and

$$r_l^{MR} = \bar{R}_l - \bar{R}_{l+1} \quad l_P \geq l \geq L-1 \qquad (22)$$

These two equations together yield another L - 1 residues.

The residue of the noise gain constraint is defined to be

$$r^{NG} = NG \sum_i A_i b_i - \sqrt{\sum_i b_i^2 \frac{\sigma_i^2}{\sigma_1^2}} \qquad (23)$$

This equation yields one additional residue.

In total there are 2*L+1 residues to express the four constraints.

The conjugate gradient technique is explained by Press et al. The termination conditions used may be an accumulative factor change in I^P of no more than 10^{-7} over 25 consecutive iterations. The total number of iterations can be limited to 3000*N, where N is the number of data points.

The one dimensional minimization problems are solved using the golden section search or parabolic interpolation described by Press et al. The 1D optimization is terminated if the functional value does not decrease by a certain fraction each step. The fraction can be set to 10^{-8}, for example. A step limit of 1000 on each 1D optimization can be set to prevent infinite or unproductive near infinite loops.

The linear transform for the trial row coefficients to the trial resolution function would usually be computed a large number of times during each one dimension optimization and would be computationally demanding. However, since the

20

relationship between the trial row coefficients and the trial resolution function is always linear, this linearity can be used to greatly reduce the number of times the trial resolution functions have to be calculated directly from the trial coefficients.

In one dimensional optimization, a scalar parameter c is varied along a line with direction d_1; the direction being provided by the conjugate gradient method. Therefore, for the trial coefficients b_i, optimization occurs along a line defined by

$$b_n = b_l^0 + c d_l^a.$$ (24)

where b^0 is also supplied by the conjugate gradient method. Because of the linearity relationship between b_i and R_1, a similar equation can be written for the corresponding trial resolution function R_1,

$$R_l = R_l^0 + c d_l^R$$ (25)

where

$$R_l^0 = \sum_i a_i^0 g_{il}$$ (26)

and

$$d_l^R = \sum_i d_i^a g_{il}.$$ (27)

Therefore, along each line of optimization, the linear transform from the trial coefficients to the trial resolution function only needs to be calculated twice, to

21

calculate R_1 and d_1, instead of once for every value of c considered. This results in a many fold reduction in the one dimensional optimization time.

The effectiveness of the minimization can be checked by
5   how close the value of the peak of the resolution function is to unity, since the minimum area should yield a peak value of exactly unity.

An upper limit on the noise gain, NG, of a particular point in an estimate can be applied by the noise gain
10   constraint stated earlier, where the area of the resolution function is included because the area of the resolution function must be normalized (set equal to 1) before the noise gain is calculated.

Figure 2 is a flow chart for calculating the
15   coefficients of a row of a transform matrix pursuant to the foregoing description, using a digital computer conventionally. Coefficients for successive rows of the matrix can be calculated in the same manner for each τ of interest and for each noise gain deemed appropriate. For
20   calculating the coefficients of any row, the coefficients of the preceding row can be used as starting points.

Constrained optimization methods may perform better if all the data have noise with a standard deviation of 1. Fortunately, it is quite simple to modify the data functions
25   so this is the case. Given the real data functions, $g_i(y)$, the adjusted data function, $g_i^\Lambda(y)$ is

$$g_i^\Lambda(y) = g_i(y)/\sigma_i \qquad (28)$$

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The adjusted data functions are then supplied to the constrained optimization method along with the assumption that the standard deviations of noise at all the data points are 1. The final coefficients, a_{ij}, produced by the

5   constrained optimization based on the adjusted data functions, need to be corrected for the adjustment. The correction to the final coefficients is

$$a_{ij} = a_{ij}^A / \sigma_j \qquad (29)$$

If a data set has 100's, 1000's, or even more evenly

.0   spaced points on a decay curve, it is much more efficient computationally for calculating the coefficients of the transform matrix (and also for applying it to data) to average adjacent data points together to create a new group data point. The corresponding data functions must also be

15   averaged together to get the grouped data function corresponding to the new grouped data points. The size of the groups is important. While, in general, it is best to have larger groups at later sample times, groups which are too large will reduce the resolution of the resolution

20   functions. Groups which are too small are inefficient. A logarithmic group size appears to be a good choice.

The equation

$$G_n = max(1, int(C \, 10^{n/D}). \qquad (30)$$

appears to produce good group sizes for C = 0.1, D = 10,

25   where l is a positive integer starting at 1 and increasing

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to whatever value is appropriate. The function max(x,y)
returns the maximum of x and y, and the function int(z)
rounds z down to the nearest integer. For 512 data points
the above equation gives group sizes of

5   1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 3 3 5 6 7 10 12 15 19 25 31 39 50 63 79 100 30

Occasionally, instead of measuring data at a point in
time on a decay curve, data will be measured by averaging
over a window between two points in time. In this case, the
data function can be approximated by averaging together a
10   larger number of sample points over the window.

Modification of the data functions will also be
required if a trigger that initiates a decay curve is not a
single point in time. For example, in time resolved
fluorescence spectroscopy, the flash of light triggering the
15   decay will last a finite length of time. If the intensity
versus time for the flash is L(t) then each data point will
be convolved with this function. The corresponding data
functions will also have to be convolved with the same
function.

20   Fig. 3 illustrates a transform matrix constructed in
accordance with the invention. Three sets of coefficients
for $\tau=1$, 10 and 100 are shown as columns so that they fit on
the page. Each set has 48 coefficients calculated from 48
data functions. The first 32 data functions correspond to
25   32 sampling points that are evenly spaced by one time unit
at times 1 to 32. The next 16 data functions are evenly

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spaced by 30 time units between time 62 and time 512. The standard deviation of the noise for the first 32 points is assumed to be 1, and the standard deviation of the noise of the next 16 is assumed to be 0.18257. The standard deviation of the noise of the last 16 is a factor of 1/sqrt(30) less than the first 32. This drop in the standard deviation of the noise would result if the cut-off frequency of a low pass filter before the analog-to-digital converter were dropped by a factor of 30 before the point was acquired. For the sake of simplicity, the matrix was constructed without the use of balancing functions or data function groups.

Fig. 4 illustrates data functions, resolution functions, noise response, and point spread functions for a matrix constructed in accordance with the invention where the noise gain is 1.000. The data function curves are for time samples at t=1, t=2 . . . t=N from left to right. Each resolution function corresponds to a set of data functions. The abscissa in each diagram is in units of τ on a log scale [ln(τ)], and each resolution function is localized about a particular τ value. As a result, each point spread function tends to be localized about a particular τ value.

Fig. 5 is a diagram similar to Fig. 4 but for a matrix constructed with a noise gain of 3.1623. It should be noted that in each of Figs. 3, 4, and 5 there are 48 data points (time samples).

25

A comparison of resolution functions produced by matrices with different noise gains is shown in Fig. 6. Higher resolution is achieved with higher noise gains, but there is a trade-off between resolution and noise gain. Greater noise tends to make the results achieved less reliable.

In addition to resolution functions, performance of a transform matrix can be judged using the PSF's and noise response. To judge the performance of a transform matrix, information is required as to corresponding time points on the decay curve at which data should be acquired as well as assumed noise at each data point.

Fig. 7 shows point spread functions associated with four matrices constructed in accordance with the invention for different noise gains. PSF's can be calculated using simulated data at required time points for monoexponential decays at τ = 1,2,5,10,20,50,100, ... time units, assuming the initial amplitude of the decay curve is 1. No noise is added to the simulated data. It should be noted that in each of Figs. 6 and 7, and also on Figs. 9 and 10 to be described, there are only 32 equally spaced data points (with the same noise).

To calculate the noise response, several realizations of a decay curve which are purely noise are generated. The noise may be assumed to be Gaussian. The standard deviation of the generated noise is scaled so that the standard deviation of the noise of the first data point is 1. Then

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the sampled noise decay curves are multiplied by the transform matrix to obtain the relaxation distribution. Several realizations of the relaxation distribution of the noise can be plotted to obtain a "feel" for the

5   distribution.

An important characteristic at each point in the estimate of the unknown model is the standard deviation due to the noise in the data. If the noise in the data is uncorrelated, has a mean of zero, and all points have the

10   same finite standard deviation, it can then be characterized by a single standard deviation. The noise gain for each point can be defined to be the standard deviation of the point divided by the standard deviation of the noise in the data and can be calculated directly from the coefficients

$$NG = \sqrt{\sum_j a_{ij}^2 \frac{\sigma_j^2}{\sigma_1^2}} \qquad (31)$$

Once a transform matrix has been constructed, experiments can be performed in which the spacing of the data acquisition points, the number of data acquisition

20   points, and the grouping of data acquisition points are changed. The results of these experiments can be compared by observing the resolution functions and/or the PSFs that are produced. As a rule of thumb, the linear resolution is proportional to the height of the resolution function

25   provided that the resolution function has unity area on a log scale.

Each trial resolution function is localized about a
point of interest in the unknown model by requiring the peak
of the resolution function to have a value greater than
unity at a τ of interest and then by minimizing the area of
5   the resolution function. The effectiveness of the
minimization can be checked by how close the value of the
peak of the resolution function is to unity, since the
minimum area should yield a peak value of unity.

From the foregoing description, it is evident that each
10   transform matrix is constructed so that each point of the
estimate of the unknown model linearly resolves the
corresponding point of the unknown model as well as possible
within an acceptable noise gain. Since matrix
multiplication is a linear operation, it yields an estimate
15   which does not necessarily reproduce the data but does have
linear resolution. With linear resolution, each point of
the estimate resolves the corresponding point of the unknown
model in the same way independent of any particular unknown
model. A highly desirable property of linear resolution in
20   providing an estimate of an unknown model is that it enables
a human interpreter to obtain an intuitive "feel" of what
the data reveals and does not reveal about the unknown
model.

Each resolution function gives a concise mathematical
25   description of the linear resolution at each point of the
unknown model and is independent of the unknown model and
the data. Viewing the transform matrix as a digital lens,

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linear combinations of the data functions are calculated

which yield a resolution function that resolves as small a

region of the unknown model as possible.

After a transform matrix is constructed and selected,

5   it is incorporated in a signal processor of a computer, as

software or hardware, for example, as indicated in Fig. 8.

In this figure the INPUT represents a source of sampled

digital signals such as multiexponential decays, e.g., NMR

decays obtained from well-logging. The DATA PROCESSOR and

10   STORAGE are components of a conventional digital computer.

The OUTPUT MODULE may have conventional DISPLAY, NETWORK

INTERFACE, and PRINTER COMPONENTS, for example.

An estimate of an unknown model is calculated by

multiplying data, e.g., multiexponential decays, by the

15   transform matrix. Several transform matrices for different

noise gains, for example, can be incorporated in the signal

processor and accessed selectively to provide a user with

greater flexibility.

As stated earlier, an object of the present invention

20   is to provided a better estimate of an unknown model, from

which useful information can be obtained. In the

aforementioned Prammer patent, Figure 8 of the patent is a

mapping of estimated NMR signal decay times into pore sizes

of an investigated earth formation. The curve of Fig. 8 is

25   an estimated relaxation distribution. The present invention

provides a better estimate of the relaxation distribution,

and also a better signal-to-noise ratio (SNR). Resolution

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functions produced by matrices constructed in accordance with the invention are superior, in terms of peak value and localization to resolution functions produced by matrices of the Prammer patent for the same data and noise.

In general, the present invention can be used to provide better estimates of an unknown model than the prior art, estimates from which more reliable and useful information can be obtained. These improved results are achieved by emphasizing linear resolution irrespective of whether data fits a model, and, in fact, without any attempt to fit data to a model. The invention is particularly useful in multiexponential signal processing, such as in the analysis of multiexponential decays.

As mentioned earlier, with particular reference to the REVIEW article, multiexponential analysis is useful in a host of scientific and technological applications. One of those applications, NMR, such as NMR in well-logging, has already been discussed. Another application is MRI (magnetic resonance imaging). Fig. 9 shows decay curves in an MRI application. The decay curves represent pixels taken from a series of 32 magnetic resonance images of the brain of a multiple sclerosis patient. The T_2 relaxation data were acquired with a 10ms sample time out to 320ms. The strength and decay of the signals contain valuable information about the tissues. Some of the major tissue types of interest in multiple sclerosis are normal appearing white matter (NAWM), cerebral spinal fluid (CSF) and

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lesions, which can be classified as chronic or acute. Fig.
10 shows relaxation distributions yielded by applying to the
decay curves of Fig. 9 transform matrices of the invention
with various noise gains. The larger the noise gain of the
estimate, the wider the range of relaxation estimates
available. This is a characteristic which shows up in the
resolution functions and is due to the fact that the larger
the noise gain the more likely there is a feasible
resolution function available.

Estimating the noise in the estimates shown in Fig. 10
can be accomplished in several ways. The first is to
estimate the noise in the data and multiply by the noise
gain for each transform matrix. The ideal way to measure
the noise in the data is to repeat the measurements a large
number of times and calculate mean, standard deviation and
covariance. These statistics can then be propagated through
transform matrices using standard statistical procedures.
Unfortunately, the measurement of the decay curves takes
about 20 minutes to complete on a patient, so large numbers
of repetitions are impractical.

Another way to measure the noise in the data is to
estimate the standard deviation from previous measurements
using the same instrument. This is not always reliable
because the noise can vary from sample to sample. For the
data in Fig. 9, previous measurements of noise gives the
standard deviation of about 10 scanner units. After
multiplying the standard deviation by the noise gain, the

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predicted standard deviation for the noise in Fig. 10 is 100.

A third way to estimate the noise is to consider that while the noise gain increases by a factor of 10 in Fig. 10, the resolution gain increases by only 1.4 for relaxation rates around five sample times. In Fig. 10, a portion of the signal between 30 and 200ms increases proportionately to the noise gain. This strongly suggests that the portion is due to random uncorrelated additive noises in the data. It is possible then to measure the standard deviation of the noise between 30 and 200ms and work back to the noise in the data.

Further applications of the invention will be apparent to persons of ordinary skill in the art. For example, decaying sinusoids are a common problem in inverse theory. A decaying sinusoid can be handled if it is band pass filtered at the particular bandwidth of interest and then the magnitude of the decay curve taken. The relaxation distribution of the magnitude decay curve can then be calculated.

Any row of a transform matrix, since it corresponds to a resolution function, can be applied to a time series in the same way as digital filters. The "relaxation" digital filter will be useful for applications such as ultrasound and radar. Using quadrature detection, it would be possible to measure the magnitude of a reflection off of an interface. If the signal from an interface of interest

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oscillated for a while after the initial sound wave had passed, the decay time of the oscillation would reveal information about the interface. Applying various relaxation digital filters to the ultrasound time series would allow characterization of the interfaces. If it were desirable to detect a particular range of relaxation time, selection of the coefficients of a relaxation digital filter would give a resolution function with a more boxcar-like shape.

Other applications of the invention include photoluminescence and time-resolved fluorescence spectroscopy of biological and other types of samples, electrical signals radiating from ore bodies in geophysical exploration and acoustic, electrical and electromagnetic decay processes. It is possible to design a transform matrix which handles data integrated over a window or unevenly spaced in time by modifying the data functions.

Principles of the invention can be applied to problems which data functions other than decay curves. If the data functions are cosine functions, resolution functions can be generated for low pass, band pass and high pass filters as well as windows for discrete Fourier transforms. A low pass filter can be designed by requiring maximum area below the cutoff frequency, minimum area above the cutoff and an optional requirement of monotonicity to eliminate wiggles. The bounds on all parts of the filter would be 0 and 1. A

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limit on broadband noise gain could also be imposed to improve the robustness of the filter.

While preferred forms of the invention have been shown and described, these forms are intended to be exemplary, not 5 restrictive, and it will be apparent to those skilled in the art that modifications can be made without departing from the principles and spirit of the invention, the scope of which is set forth in the appended claims.

WHAT IS CLAIMED IS:

1.   A computer-readable medium containing a transform operator constructed to provide an estimate of an unknown model with substantially optimal linear resolution.

2.   A computer-readable medium according to Claim 1, wherein the transform operator comprises a matrix having at least one row of coefficients corresponding to a resolution function.

3.   A computer-readable medium according to Claim 2, wherein the resolution function has substantially no negative values.

4.   A computer-readable medium according to Claim 2, wherein the resolution function has a peak value of at least substantially 1 substantially at the point of interest.

5.   A computer-readable medium according to Claim 2, wherein the resolution function monotonically decreases from its peak value.

6.   A computer-readable medium according to Claim 2, wherein the resolution function has substantially no negative values, has a peak value of at least substantially 1 substantially at the point of interest, and decreases substantially monotonically from its peak value.

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7. A computer-readable medium according to Claim 6, wherein the resolution function complies substantially with the following noise gain constraint:

$$\sqrt{\sum_{i=1}^{N} b_i^2 \frac{\sigma_i^2}{\sigma_1^2}} \leq NG \int_{-\infty}^{+\infty} \sum_{i=1}^{N} b_i g_i(y) dy \quad \text{[Noise gain constraint]}$$

8. A computer-readable medium according to Claim 2, wherein the matrix has a plurality of rows of coefficients, each row corresponding to a different resolution function.

9. A computer-readable medium according to Claim 8, wherein the matrix is constructed for use in multiexponential decay signal processing and wherein each resolution function is substantially centered on a different time constant.

10. A computer-readable medium according to Claim 9, wherein each coefficient has a corresponding data function in the form $e^{-t_k e^{-y}}$ or derivations of this form, where y is the natural logarithm of a time constant and t_k is a sampling point on the decay signal.

11. A computer-readable medium according to Claim 2, wherein the coefficients are selected so that the corresponding resolution function has a peak value of at least substantially 1 substantially at the point of interest and wherein the area of the resolution function is substantially minimized.

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12.   A computer-readable medium according to Claim 11,
wherein the area of the resolution function is set to be
substantially 1 on a logarithmic scale.

13.   A method of constructing a transform operator in
which a plurality of coefficients are selected to produce a
corresponding resolution function that provides
substantially optimal linear resolution.

14.   A method according to Claim 13, wherein the
resolution function has substantially no negative values,
has a peak value of at least substantially 1 substantially
at the point of interest, decreases substantially
monotonically from the peak value, and has a gain within a
predetermined range.

15.   A method according to Claim 13, wherein the area
of the resolution function is set to be substantially 1 on a
logarithmic scale.

16.   A method according to Claim 13, wherein the
transform operator is constructed to provide substantially
optimal linear resolution between data and an unknown model
without any consideration of whether the data fits the
unknown model.

17. A method of multiexponential signal processing,
which comprises:

sampling a multiexponential signal and applying the
transform operator of Claim 1 to the sampled signal.

18. Apparatus for multiexponential signal processing,
which comprises a signal processor that has the transform
operator of Claim 1.

19. A method of constructing a transform operator, in
which a plurality of coefficients are calculated using data
functions that emphasize linear resolution irrespective of
data.

20. A method according to Claim 19, wherein each data
function is in the form $e^{-t_k e^{-y}}$ or derivations of this form,
where y is the natural logarithm of a time constant and t_k
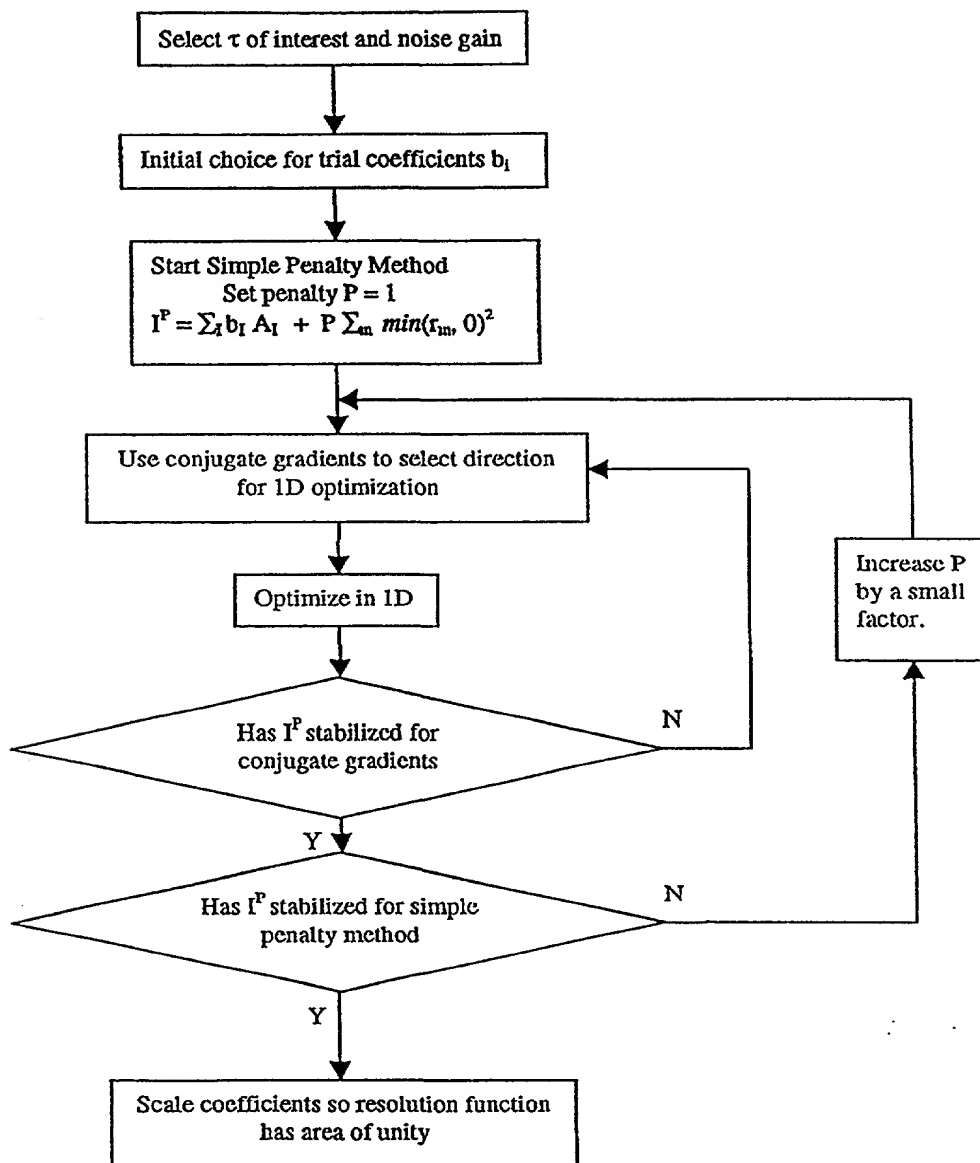is a sampling point on a decay signal.

21. A method of exponential signal processing, which
comprises:
providing a sampled multiexponential signal; and
applying the sampled multiexponential signal to a
transform operator constructed to provide substantially
optimal linear resolution between outputs of the transform
operator and an unknown model.

ABSTRACT OF THE DISCLOSURE

For signal processing of transients such as multiexponential decays, a transform operator (e.g., matrix) is constructed by emphasizing linear resolution, rather than using fitting routines that attempt to find an estimate of an unknown model that fits data. Use of the transform operator to process multiexponential signals produces outputs that are a better estimate of the unknown model or of some segment of the unknown model.

$$
\begin{pmatrix}
a_{11} & a_{21} & \ldots & a_{N1} \\
a_{12} & a_{22} & \ldots & a_{N2} \\
a_{13} & a_{23} & & a_{N3} \\
\vdots & & & \vdots \\
a_{1K} & a_{2K} & \ldots & a_{NK}
\end{pmatrix}
\begin{pmatrix}
d_1 \\
d_2 \\
\vdots \\
d_N
\end{pmatrix}
=
\begin{pmatrix}
m_1 \\
m_2 \\
m_3 \\
\vdots \\
m_K
\end{pmatrix}
$$

FIG. 1

Select τ of interest and noise gain

↓

Initial choice for trial coefficients b_i

↓

Start Simple Penalty Method
Set penalty P = 1
$I^P = \sum_I b_I A_I + P \sum_m min(r_{m}, 0)^2$

↓

Use conjugate gradients to select direction
for 1D optimization

↓

Optimize in 1D

↓

Has I^P stabilized for
conjugate gradients ──N──→

Increase P
by a small
factor.

Y ↓

Has I^P stabilized for simple
penalty method ──N──→

Y ↓

Scale coefficients so resolution function
has area of unity

**Flow Chart for calculating the coefficients of a row
of a transform matrix**

FIG. 2

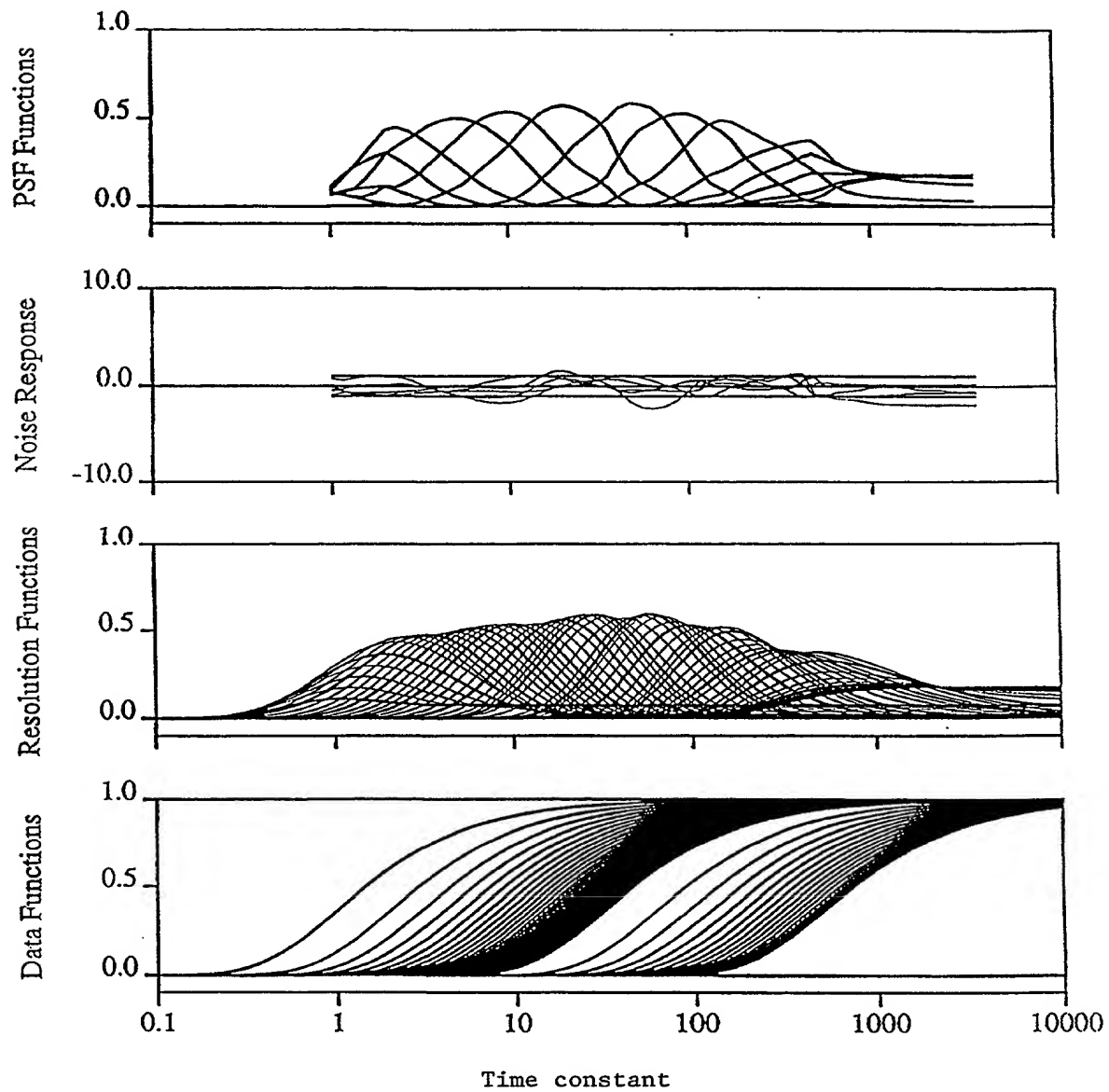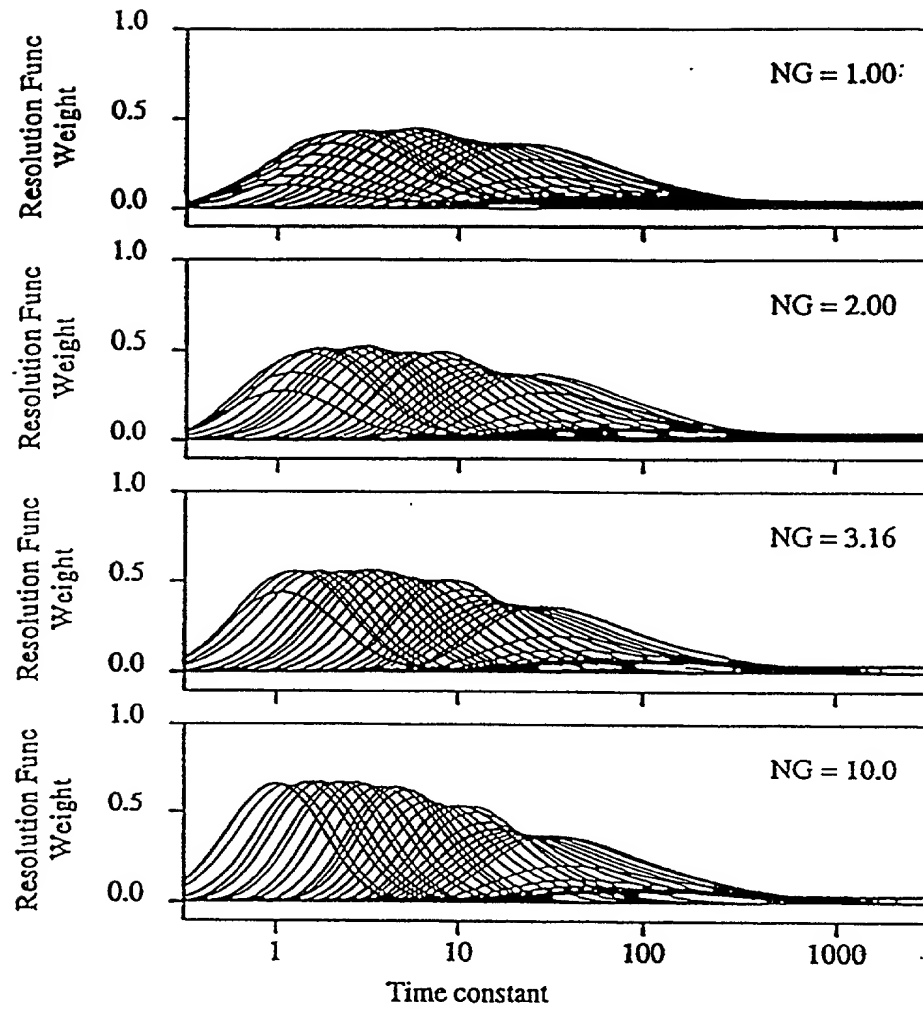| Data Function	Time Point	Noise	Tau Time		
			1.00000000	10.00000000	100.00000000
1	1	1.0000	0.57510339	0.00749061	-0.00000022
2	2	1.0000	-0.66730933	-0.02936037	0.00116325
3	3	1.0000	-0.25419531	-0.01416762	-0.00103054
4	4	1.0000	0.10842690	0.11491226	-0.00517314
5	5	1.0000	0.22891866	0.21555085	0.00150016
6	6	1.0000	0.20253408	0.26374170	0.00768891
7	7	1.0000	0.12323711	0.27121720	-0.00815470
8	8	1.0000	0.04259521	0.25261365	0.00375738
9	9	1.0000	-0.01896607	0.21886739	-0.00258380
10	10	1.0000	-0.05746454	0.17728047	-0.00824487
11	11	1.0000	-0.07590029	0.13257193	-0.01166370
12	12	1.0000	-0.07936233	0.08774856	-0.01231092
13	13	1.0000	-0.07285695	0.04469294	-0.01039074
14	14	1.0000	-0.06052232	0.00454447	-0.00652068
15	15	1.0000	-0.04547436	-0.03205290	-0.00147989
16	16	1.0000	-0.02990452	-0.06478717	0.00395850
17	17	1.0000	-0.01525442	-0.09356850	0.00912369
18	18	1.0000	-0.00239347	-0.11845430	0.01349171
19	19	1.0000	0.00822711	-0.13959775	0.01669554
20	20	1.0000	0.01645044	-0.15721183	0.01851551
21	21	1.0000	0.02231823	-0.17154437	0.01885886
22	22	1.0000	0.02599914	-0.18286058	0.01773536
23	23	1.0000	0.02773615	-0.19143096	0.01523274
24	24	1.0000	0.02780899	-0.19752293	0.01149409
25	25	1.0000	0.02650790	-0.20139530	0.00669831
26	26	1.0000	0.02411594	-0.20329460	0.00104399
27	27	1.0000	0.02089771	-0.20345286	-0.00526337
28	28	1.0000	0.01709265	-0.20208637	-0.01202120
29	29	1.0000	0.01291166	-0.19939516	-0.01903681
30	30	1.0000	0.00853608	-0.19556306	-0.02613247
31	31	1.0000	0.00411827	-0.19075802	-0.03314860
32	32	1.0000	-0.00021688	-0.18513272	-0.03994516
33	62	0.1825	-0.19819362	0.37951780	-0.28497271
34	92	0.1825	0.23267523	1.33318197	2.49240606
35	122	0.1825	-0.04509115	0.49801344	1.56412931
36	152	0.1825	-0.10738803	-0.24390701	-0.32155463
37	182	0.1825	-0.01883294	-0.54487394	-1.65180794
38	212	0.1825	0.05055656	-0.50315444	-2.11140914
39	242	0.1825	0.05490210	-0.29078933	-1.85286598
40	272	0.1825	0.01981092	-0.04695913	-1.15211585
41	302	0.1825	-0.01807900	0.14287372	-0.27784002
42	332	0.1825	-0.03647801	0.24100699	0.55035519
43	362	0.1825	-0.03064447	0.24498051	1.16899672
44	392	0.1825	-0.00822753	0.17533224	1.46502828
45	422	0.1825	0.01707418	0.06568403	1.36638935
46	452	0.1825	0.03041819	-0.04456328	0.83252224
47	482	0.1825	0.01882868	-0.11525335	-0.15387804
48	512	0.1825	-0.02735607	-0.10869726	-1.59354436

FIG. 3

FIG. 4

FIG. 5

Time constant

FIG 6

FIG. 7

FIG. 8

INPUT

DATA
PROCESSOR

DISPLAY

NETWORK
INTERFACE

STORAGE

SET OF
TRANSFORM
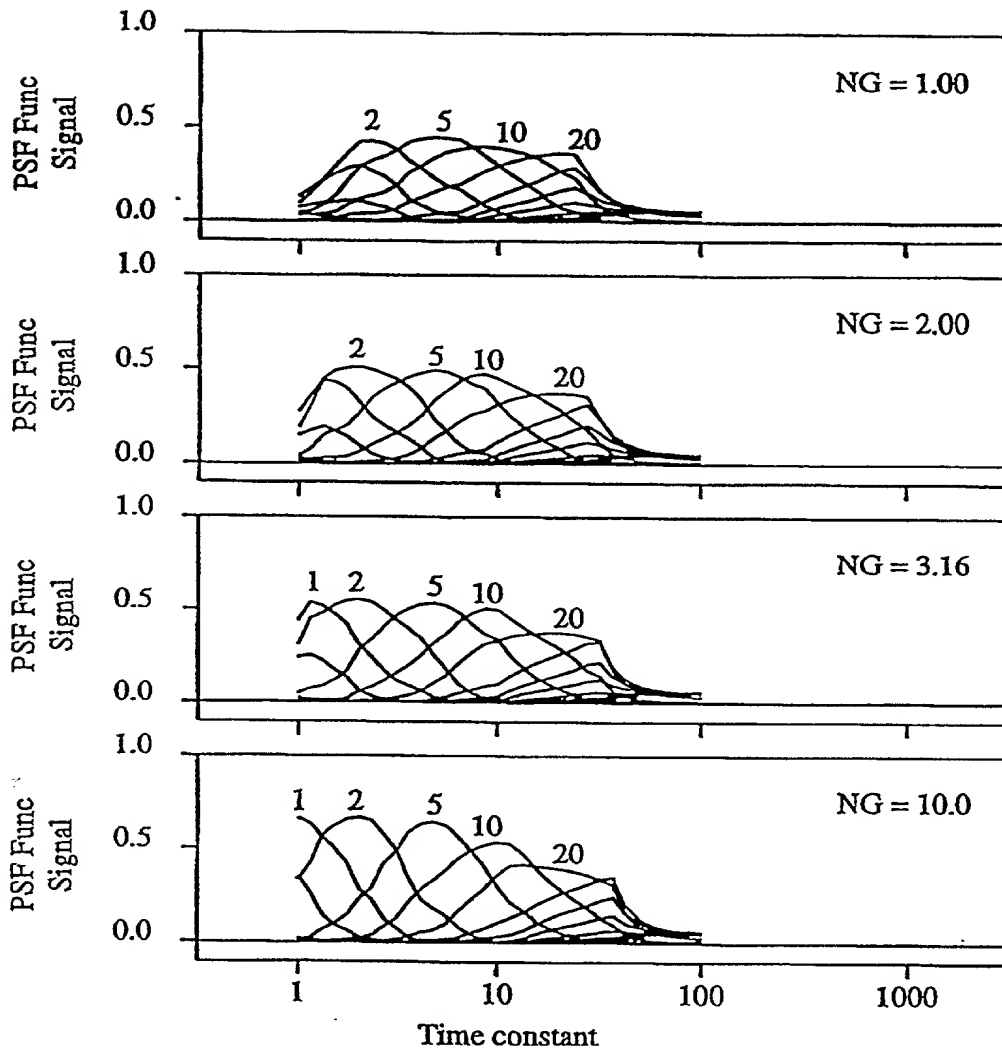COEFFICIENTS
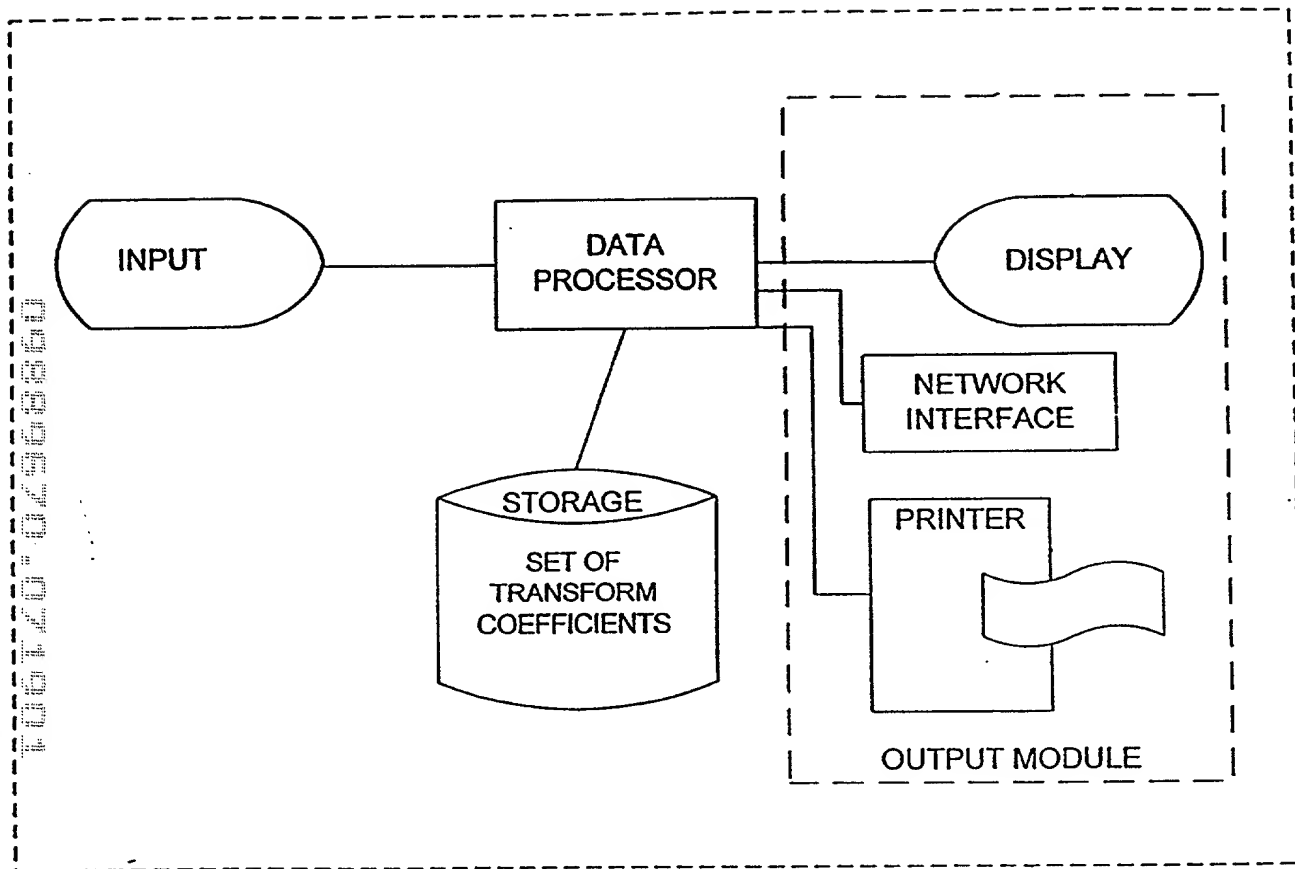
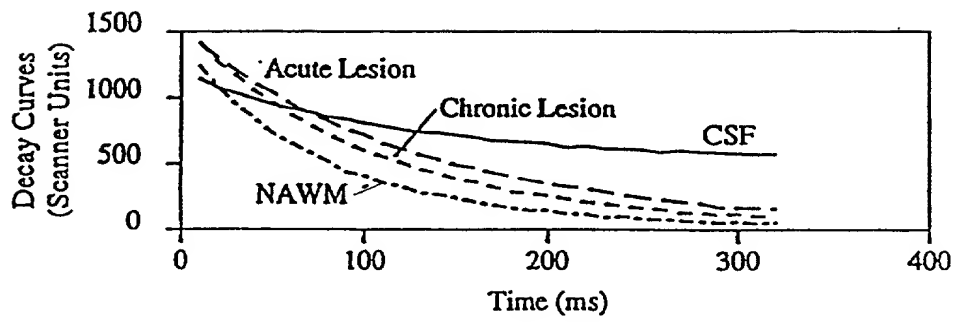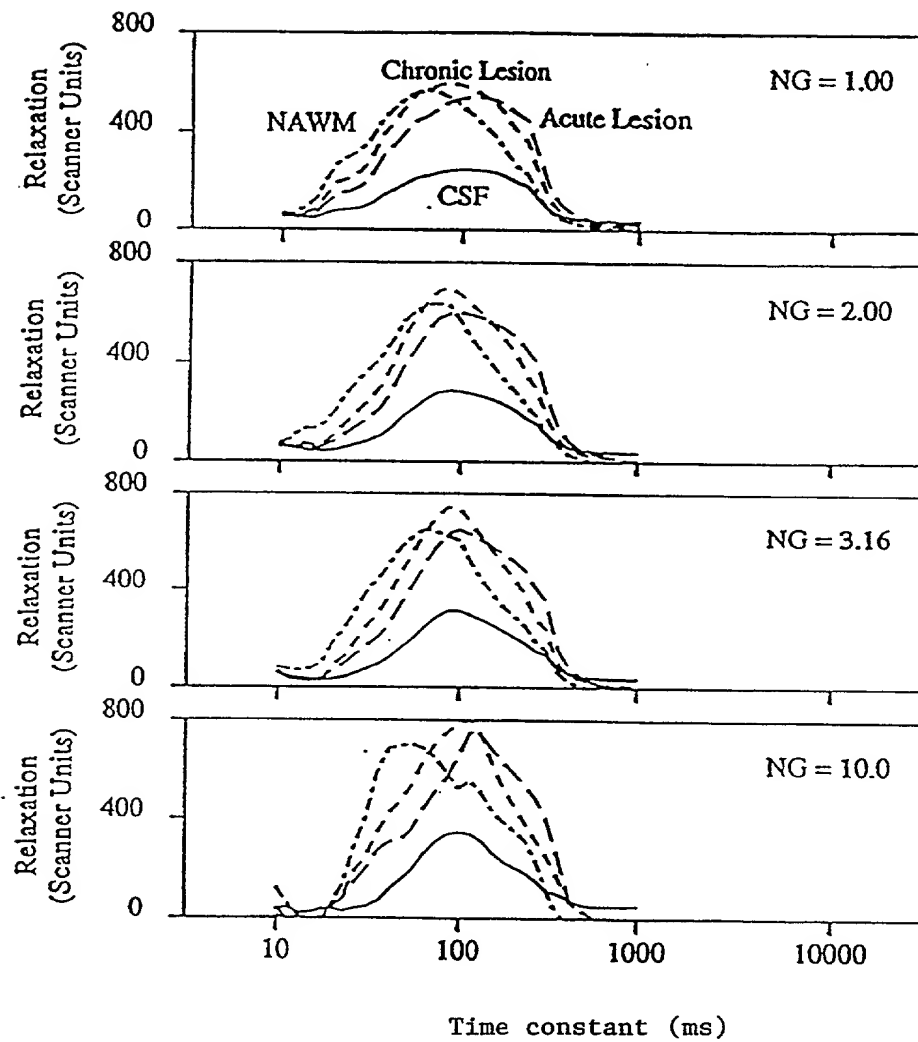PRINTER

OUTPUT MODULE

FIG. 9

FIG. 10

Time constant (ms)
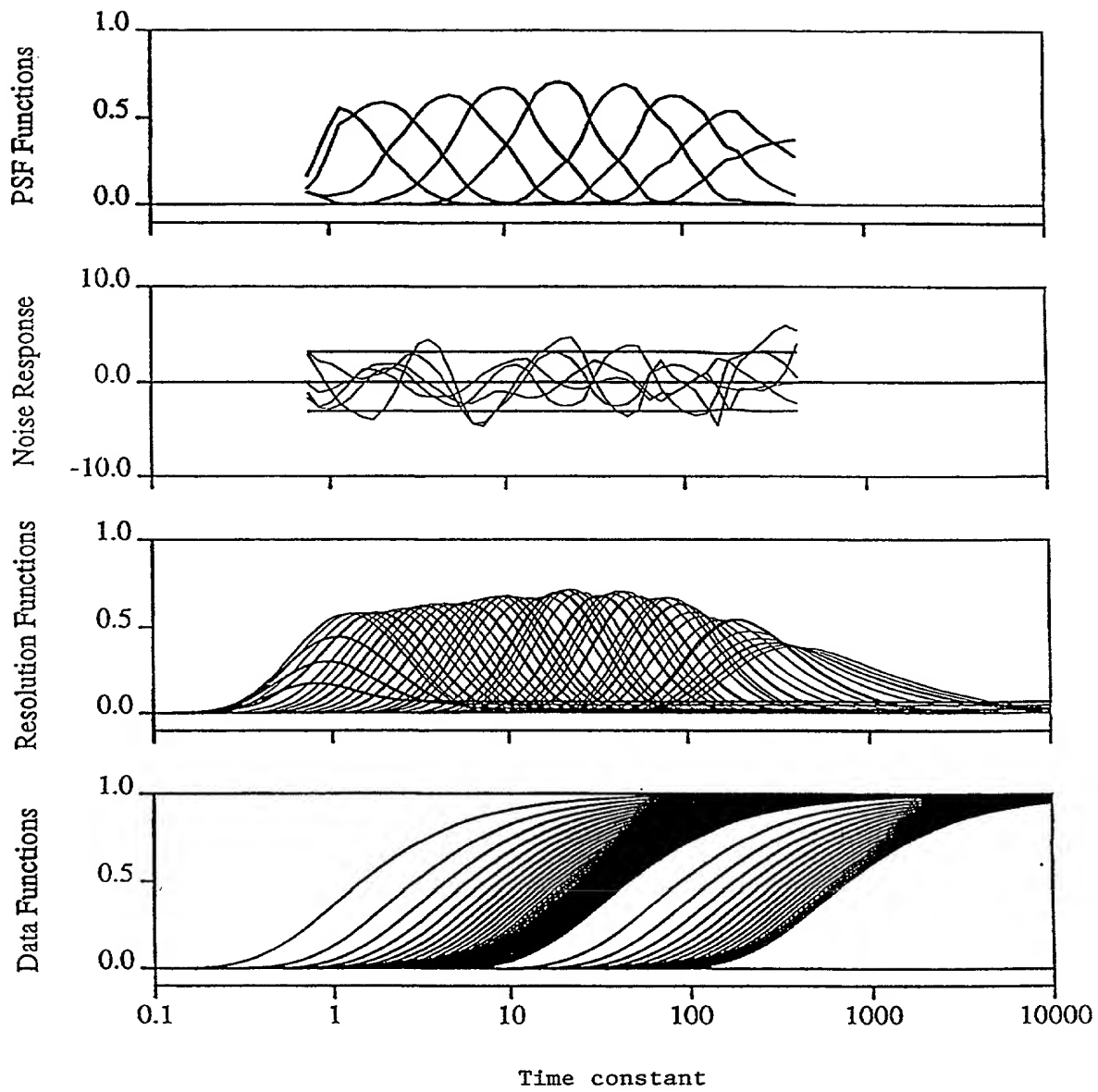
DECLARATION AND POWER OF ATTORNEY

As a below-named inventor, I hereby declare that:

My residence, post office address and citizenship are as stated below next to my name.

I believe that I am the original, first and sole inventor (if only one name is listed below) or an original, first and joint inventor (if plural names are listed below) of the subject matter which is claimed and for which a patent is sought on the invention entitled __Multiexponential Signal Processing Method and Apparatus Using Linear Resolution__

the specification of which

☐   is attached hereto.

☒   was filed on __January 19, 2000__, as United States Application No. _____
or PCT International Application No. __PCT/IB00/00212__, and was amended on _____

I hereby state that I have reviewed and understand the contents of the above-identified specification, including the claims, as amended by any amendment referred to above. I acknowledge the duty to disclose information which is material to patentability as defined in Title 37, Code of Federal Regulations, § 1.56.

I hereby claim foreign priority benefits under Title 35, United States Code, § 119 (a)-(d) or (f), or 365(b) of any foreign application(s) for patent, inventor's or plant breeder's rights certificate(s), or 365(a) of any PCT international application which designated at least one country other than the United States of America, listed below and have also identified below any foreign application for patent, inventor's or plant breeder's rights certificate(s), or any PCT international application having a filing date before that of the application on which priority is claimed:

			Priority Claimed	
Application No.	Country	Filing Date	Yes	No
			☐	☐
			☐	☐
			☐	☐
			☐	☐

I hereby claim the benefit under Title 35, United States Code, § 119(e) of any United States provisional application(s) listed below:

Application No.	Filing Date
60/116,362	January 19, 1999

I hereby claim the benefit under Title 35, United States Code, § 120 of any United States application(s) listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of Title 35, United States Code, § 112, I acknowledge the duty to disclose information which is material to patentability as defined in Title 37, Code of Federal Regulations, § 1.56 which became available between the filing date of the prior application and the national or PCT international filing date of this application:

Application No.	Filing Date

I hereby appoint Nelson H. Shapiro, Reg. No 17,095, Mitchell W. Shapiro, Reg. No. 31,568, and the other practitioners associated with the Customer Number provided below to prosecute this application and to transact all business in the Patent and Trademark Office connected therewith, and direct that all correspondence be addressed to that Customer Number:

Customer Number 000181.

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under § 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

(1)   Full name of sole or first inventor: Keith Cover

Date: __July 19, 2001__   Signature: _____

Citizenship: __Canada__   Residence: __Vancouver, British Columbia, Canada__

Post Office Address: __Dept. of Physics and Astronomy, 6224 Agricultural Rd., Vancouver, British Columbia V6T 1Z1 Canada__

(2)   Full name of second joint inventor, if any: _____

Date: _____   Signature: _____

Citizenship: _____   Residence: _____

Post Office Address: _____